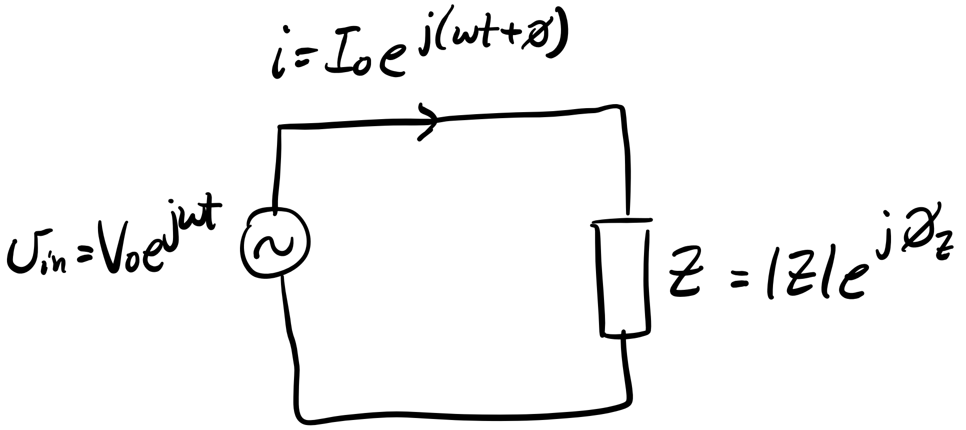
PHYS 231 - Oct. 25, 2023

<u>Last Time</u>: For any circuit of the form:

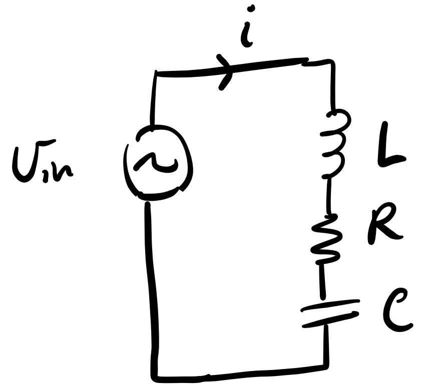


$$I_0 = \frac{V_0}{|Z|} \qquad \phi = -\phi_Z = -\tan^{-1}\left(\frac{Im[Z]}{Re[Z]}\right)$$

For a capacitor $Z_C = \frac{1}{j\omega C}$

For an inductor $Z_L = j\omega L$

Series LRC circuit



$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \qquad \tan\phi_Z = \frac{\omega L}{R} - \frac{1}{\omega RC}$$

$$\Rightarrow \quad I_0 = \frac{V_0/R}{\sqrt{1+\left(\omega\frac{L}{R} - \frac{1}{\omega RC}\right)^2}}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega RC} - \omega\frac{L}{R}\right)$$

In Exp. #4 day 2, you will meas. the volt. across the resistor as a fcn of the signal generator freq. f $(\omega = 2\pi f)$.

$$v_R = iR$$

Amplitude of v_R is just $V_R = I_0 R$

$$V_R = \frac{V_0}{\sqrt{1+\left(\omega\frac{L}{R} - \frac{1}{\omega RC}\right)^2}}$$

the phase of the resistor voltage is the same as the current phase

$$\phi = \tan^{-1}\left(\frac{1}{\omega RC} - \omega\frac{L}{R}\right)$$

In Exp #4 day 2, you will complete an analysis of

$$\frac{V_R}{V_0} \text{ vs } \omega$$

in the provided Jupyter notebook.

Expect $$\frac{V_R}{V_0} = \frac{1}{\sqrt{1+\left(\omega\frac{L}{R} - \frac{1}{\omega RC}\right)^2}}$$

$$\phi = \tan^{-1}\left(\frac{1}{\omega RC} - \omega\frac{L}{R}\right)$$

Check limiting behaviours...

1. $\omega \to 0$ (small)

$$\omega\frac{L}{R} - \frac{1}{\omega RC} \approx -\frac{1}{\omega RC}$$

$$1+\left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)^2 \approx \frac{1}{(\omega RC)^2}$$

$$\frac{V_R}{V_0} \approx \frac{1}{\frac{1}{\omega RC}} \approx \omega RC \rightarrow \text{linear}$$

$$\phi \approx \tan^{-1}\left(\frac{1}{\omega RC}\right) \rightarrow \tan^{-1}(\infty)$$

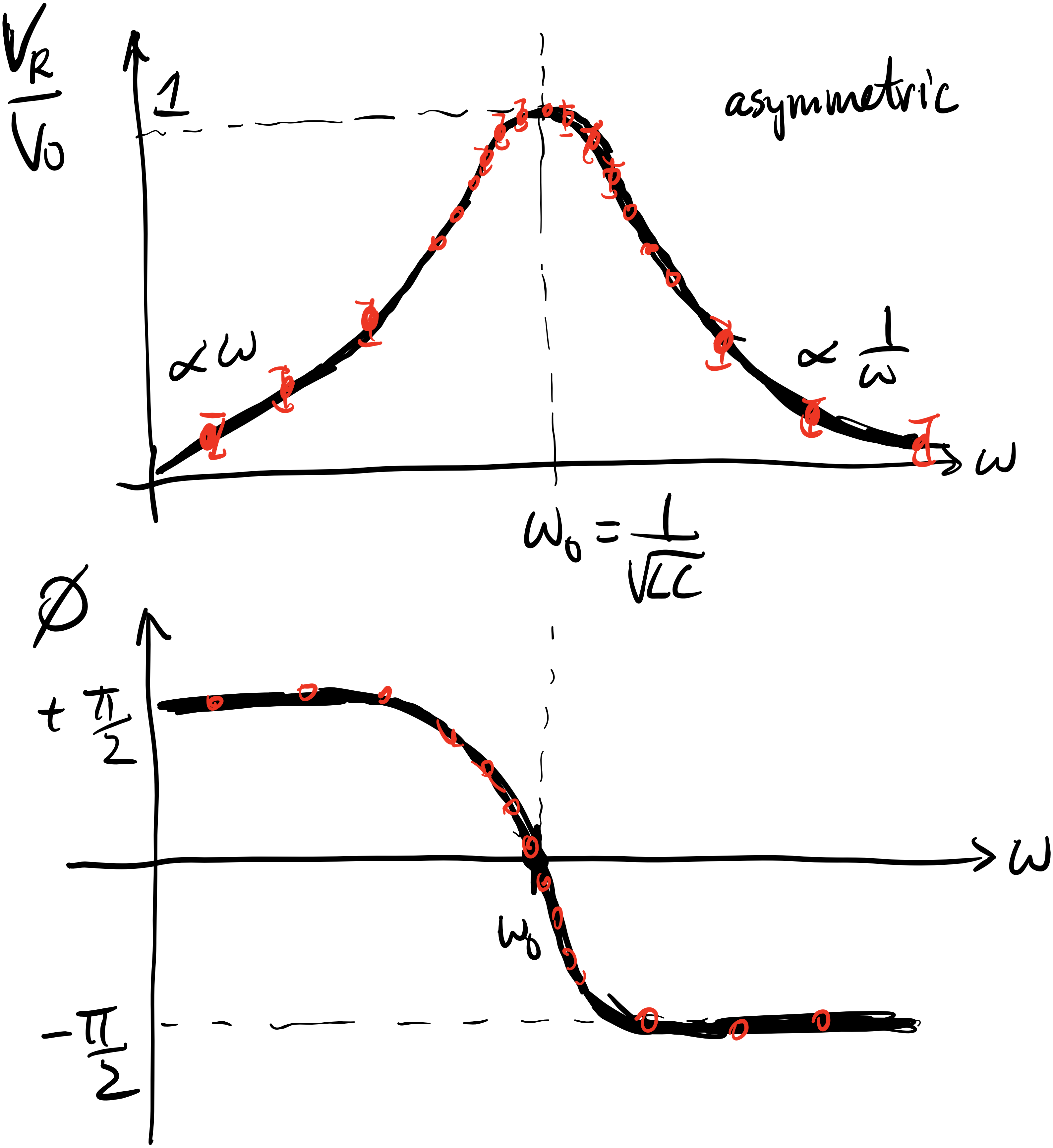
$\therefore \phi \rightarrow +\pi/2$ at low freq.

2. $\omega \rightarrow \infty$ (large) $\quad \omega\frac{L}{R} - \frac{1}{\omega RC} \approx \omega\frac{L}{R}$

$$1 + \left(\omega\frac{L}{R} - \frac{1}{\omega RC}\right)^2 \approx \left(\omega\frac{L}{R}\right)^2$$

$$\frac{V_R}{V_0} \approx \frac{1}{\omega\frac{L}{R}} = \frac{1}{\omega}\frac{R}{L} \propto \frac{1}{\omega}$$

$$\phi \approx \tan^{-1}\left(-\omega\frac{L}{R}\right) \longrightarrow -\frac{\pi}{2}$$ at high freq.



$\frac{V_R}{V_0}$
1
asymmetric
$\propto \omega$
$\propto \frac{1}{\omega}$
ω
$\omega_0 = \frac{1}{\sqrt{LC}}$
ϕ
$+\frac{\pi}{2}$
ω
ω_0
$-\frac{\pi}{2}$

Resonance Frequency.

$$\frac{V_R}{V_0} = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R} - \frac{1}{\omega RC}\right)^2}}$$ is a

maximum when $\omega = \omega_0$ s.t.

$$\omega_0 \frac{L}{\cancel{R}} - \frac{1}{\omega_0 \cancel{R} C} = 0$$

$$\Rightarrow \omega_0^2 = \frac{1}{LC} \Rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

when $\omega = \omega_0$, $\frac{V_R}{V_0} = 1$

Phase at $\omega = \omega_0$:

$$\phi = \tan^{-1}\left(\frac{1}{\omega_0 RC} - \frac{\omega_0 L}{R}\right)$$

$$= \tan^{-1}\left[\frac{1}{R}\underbrace{\left(\frac{1}{\omega_0 C} - \omega_0 L\right)}\right]$$

$$\frac{1}{\frac{C}{\sqrt{LC}}} - \frac{1}{\sqrt{LC}} L$$

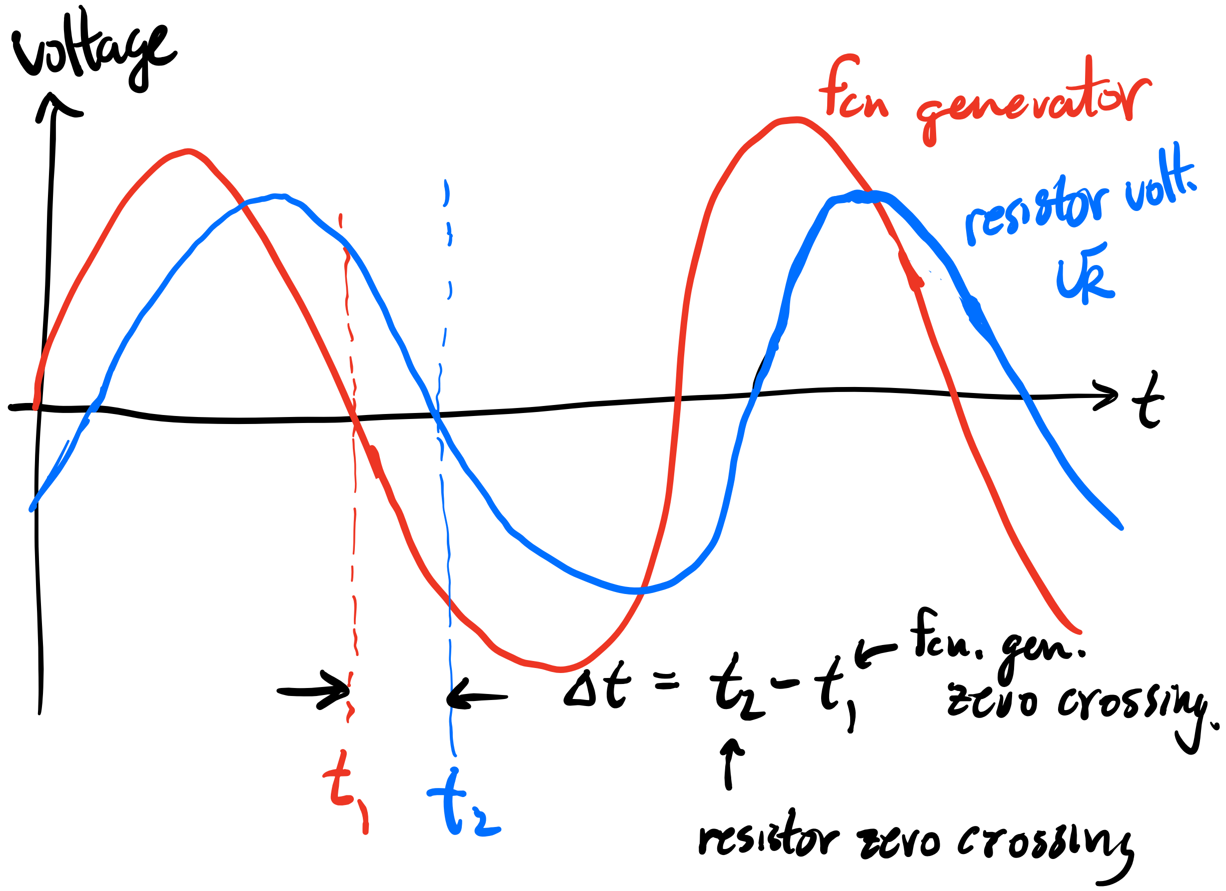
$$= \frac{1}{\sqrt{\frac{C}{L}}} - \sqrt{\frac{L}{C}}$$

$$= \sqrt{\frac{L}{C}} - \sqrt{\frac{L}{C}} = 0$$

$$\therefore \phi = \tan^{-1}(0) = 0$$

At resonance ($\omega = \omega_0$), the resistor voltage V_R is in phase ($\phi = 0$) with the output of the signal generator.

Measuring Phase differences using the oscilloscope.



Convert Δt to a phase difference ϕ

$$\frac{\phi}{\Delta t} = \frac{-\pi}{T/2}$$

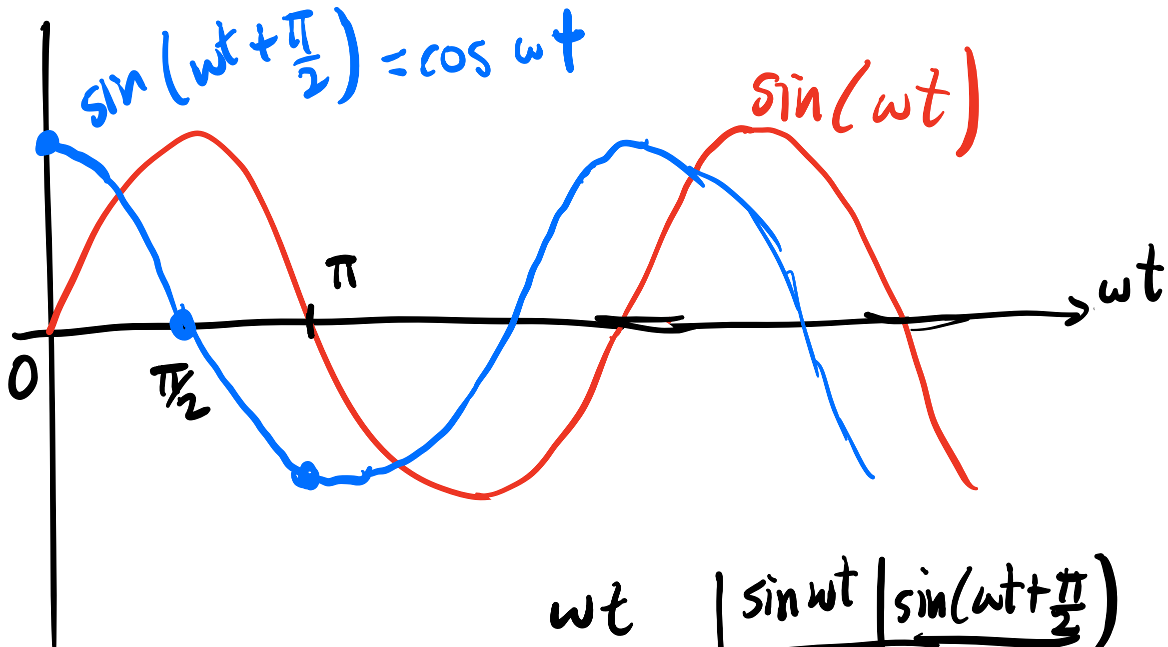
(half period shift in time corresponds to a phase shift of π)

$$\therefore \varnothing = -2\pi \frac{\Delta t}{T} = -(2\pi f)\Delta t$$

to determine $\varnothing$, meas. Δt and the calc.

$$\boxed{\varnothing = -2\pi f \Delta t}$$

Sign of the Phase.



$$f = \sin\left(\omega t + \frac{\pi}{2}\right)$$

ωt	$\sin \omega t$	$\sin(\omega t + \frac{\pi}{2})$
0	0	$\sin \frac{\pi}{2} = 1$
$\pi/2$	1	$\sin \pi = 0$
π	0	$\sin \frac{3\pi}{2} = -1$